1. **A generalization of Donsker’s theorem.** [5 points]

Suppose \( \xi_{n,i}, 1 \leq i \leq k_n \), is a row-wise independent triangular array of random variables with \( \mathbb{E}\xi_{n,i} = 0, \mathbb{E}\xi_{n,i}^2 = \sigma_{n,i}^2 \). Let \( S_{n,i} = \sum_{j \leq i} \xi_{n,j}, s_{n,i}^2 = \sum_{j \leq i} \sigma_{n,j}^2, s_n^2 = s_{n,k_n}^2 \). Define a random function \( X_n \) by specifying \( X_n(\frac{s_{n,i}}{s_n}) = \frac{S_{n,i}}{s_n} \) and then linearly interpolating on \([\frac{s_{n,i}}{s_n}, \frac{s_{n,i+1}}{s_n}]\), \( 0 \leq i < k_n \).

Assume the following two conditions:

(a) \( \frac{1}{s_n^2} \sum_{i=1}^{k_n} \int \xi_{n,i}^2 1_{\{|\xi_{n,i}| > ts_n\}} \to 0 \) for any \( t > 0 \).

(b) \( \max_{1 \leq i \leq k_n} \frac{s_{n,i}^2}{s_n^2} \to 0 \).

Show then that \( X_n \stackrel{w}{\to} B \). (You may only indicate the modifications required in the proof of the i.i.d. case.)

2. **Borel measurability of \( C[0, 1] \).** [5 points]

Is \( C[0, 1] \) a Borel subset of \((D[0, 1], \mathcal{D})\)?

3. **Subspace topology on \( C[0, 1] \) inherited from the Skorohod topology.** [5 points]

Show that the subspace topology on \( C[0, 1] \) inherited from \((D[0, 1], \mathcal{D})\) is the usual uniform topology.

4. **\( L_1 \) topology on \( D[0, 1] \).** [5 points]

Show that \( \rho(x, y) = \int_0^1 |x(t) - y(t)| \, dt \) is a metric on \( D[0, 1] \). Show that the Skorohod topology is strictly finer than the \( L_1 \) topology induced by \( \rho \) (i.e. open sets in the latter topology are also open in the former but some open set in the former is not open in the latter).