1. **Consistency of sample quantiles.**

   Let $Q_F(\alpha) = \inf\{x \mid F(x) \geq \alpha\}$ be the $\alpha$-th quantile of a CDF $F$, $\alpha \in (0, 1)$. Suppose that $F(x) > \alpha$ for all $x > Q_F(\alpha)$. Let $\hat{F}_n$ be the empirical CDF of an i.i.d. sample from $F$. Assuming the classical univariate Glivenko-Cantelli theorem, show that the $\alpha$-th sample quantile $Q_{\hat{F}_n}(\alpha)$ is a consistent estimator of $Q_F(\alpha)$.

2. **VC dimension of various Boolean function classes.**

   Compute the VC dimensions of the following Boolean function classes:

   (a) $\mathcal{F}_1 = \{1_{(-\infty, t]} \mid t \in \mathbb{R}^d\}$.
   (b) $\mathcal{F}_2 = \{f_t : [-1, 1] \to \mathbb{R} \mid t \in \mathbb{R}\}$, where $f_t(x) = \text{sign}(\sin(tx))$.

3. **Mean and variance of sub-Gaussian variables.**

   Suppose that $X$ satisfies
   \[
   \mathbb{E}e^{\lambda X} \leq e^{\mu t + \sigma^2 t^2} \quad \text{for all } \lambda \in \mathbb{R}.
   \]
   Prove or disprove the following:

   (a) $\mathbb{E}X = \mu$.
   (b) $\text{Var}(X) \leq \sigma^2$.
   (c) Let $\sigma^2_\star$ be the smallest possible $\sigma^2$ such that $(\star)$ holds. Then $\text{Var}(X) = \sigma^2_\star$. 